DAI-003-2014008 Seat No. ______

B. Sc. (Sem. IV) (W.E.F. 2019) (CBCS) Examination

April - 2022

Mathematics : Paper - IV (A) Theory

(Linear Algebra, Real Analysis & Differential Geometry)

Faculty Code : 003
Subject Code : 2014008

Time : $2\frac{1}{2}$ Hours] [Total Marks : **70**

Instructions : (1) All five questions are compulsory.
(2) Figures to the right indicate marks of corresponding question.

1 (a) Answer the question briefly : **4**

(1) Is the sequence $\left\{\frac{n!}{n^n}\right\}$ convergent/divergent?

(2) Define bounded above sequence.

(3) Every convergent sequence has more than one limit.
(True/False)

(4) Verify the sequence $\left\{1+\frac{(-1)^n}{n}\right\}$ is convergent.

(b) Answer any one of the following : **2**

(1) Discuss the convergence of sequence $\left\{\frac{4^{3n}}{3^{4n}}\right\}$.

(2) Show that every convergent sequence is bounded.

(c) Answer any one of the following : **3**

(1) Discuss the convergence of the sequence $\left\{\left(\sqrt{n+1}-\sqrt{n}\right)\right\}$.

(2) Prove that $\lim_{n\to\infty} \sqrt[n]{n=1}$

(d) Answer any one of the following : **5**

(1) If $\{a_n\}, \{b_n\}$ be two sequences such that $\lim_{n\to\infty} a_n = a$ and $\lim_{n\to\infty} b_n = b$ then prove that $\lim_{n\to\infty}\left(a_n + b_n\right) = a + b$.

(2) Discuss the convergence of the sequence $S_1 = \sqrt{2}$, $S_{n+1} = \sqrt{2+S_n}, \forall\, n \in N$ and find its limit if exists.

2 (a) Answer the question briefly : **4**

(1) Define D'Alembert's Ratio Test.

(2) If $\lim_{n\to\infty} u_n = 0$ then $\sum u_n$ is convergent?

(True / False)

(3) Can we use the ratio test to determine the series $\sum \frac{1}{n^3}$ is convergent ?

(4) Is the series $\sum \frac{1}{n^{3/2}}$ convergent ? Why ?

(b) Answer any one of the following : **2**

(1) Discuss convergence of the series $\sum \frac{1}{n(n+1)}$ by practical comparison test.

(2) Discuss convergence of the series $\sum \frac{1}{2^n n^2}$.

(c) Answer any one of the following : **3**

(1) Find the radius of convergence of the series $\sum_{n=1}^{\infty} \frac{(-1)^n x^n}{3^n}$.

(2) Discuss the convergence of the series $\frac{1}{1.2}+\frac{2}{3.4}+\frac{3}{5.6}+........$

(d) Answer any one of the following : **5**

(1) Discuss the convergence of the series $\frac{1}{2}+\frac{1.3}{2.4}+\frac{1.3.5}{2.4.6}+.......$

(2) Test the convergence of the series

$$1+\frac{x}{2}+\frac{x^2}{5}+\frac{x^3}{10}+......+\frac{x^n}{n^2+1}+......$$

3 (a) Answer the question briefly : **4**

(1) Define linear transformation.

(2) Define kernel of linear transformation.

(3) For linear transformation $T = R^2 \to R^3$, defined as $T(x, y) = (x, x+y, y), \quad \forall (x, y) \in R^2$, then find kernel of T.

(4) State Rank-Nullity Theorem.

(b) Answer any one of the following : **2**

(1) Show that $T: R^2 \to R^2$, defined as $T(x, y) = (x+1, y-2)$, $\forall (x, y) \in R^2$, is not linear transformation.

(2) Find the Linear Transformation $T: R^3 \to R^3$, such that $R(T) = SP\{(1, 5, 0), (0, 7, 3)\}$.

(c) Answer any one of the following : **3**

(1) Find linear transformation $T: R^3 \to R^2$, such that $T(e_1) = (1, 1)$, $T(e_1 + e_2) = (1, 0)$, $T(e_1 + e_2 + e_3) = (1, -1)$. Also find $T(2, 5, 7)$, where $\{e_1, e_1 + e_2, e_1 + e_2 + e_3\}$ is a basis of R^3.

(2) A linear mapping $T: R^3 \to R^2$ such that $T(x, y, z) = (x - y + z, x + y - z)$, $\forall (x, y, z) \in R^3$, then find $N(T), n(T), R(T), r(T)T$.

(d) Answer any one of the following :

(1) Let $T: R^3 \to R^3$, is such that $T(a, b, c) = (a - b + c, b - c, c)$, $\forall (a, b, c) \in R^3$, then find T^{-1}, if exist.

(2) Prove that a linear transformation $T: U \to V$ is one-one if and only if $N_T = \{\theta\}$.

4 (a) Answer the question briefly : **4**

(1) Define Dual of vector space.

(2) Define Eigen vector of a linear transformation.

(3) Define Adjoint of linear transformation.

(4) Find the Eigen value for corresponding matrix $A = \begin{bmatrix} 1 & 1 \\ -1 & 3 \end{bmatrix}$.

(b) Answer any one of the following : **2**

(1) Find the Eigen value, vector for corresponding matrix $A = \begin{bmatrix} 1 & 4 \\ 3 & 2 \end{bmatrix}$.

(2) Let $T : R^2 \to R^2$, $T(x, y) = (x\cos\theta + y\sin\theta, x\sin\theta + y\cos\theta)$, $\forall (x, y) \in R^2$ and, B standard base of R^2 then find $[T; B]$.

(c) Answer any one of the following : **3**

(1) $B_1 = \{1, x, x^2\}$, $B_2 = \{1, x, x^2, x^3\}$ are the bases of $P_2(R), P_3(R)$ respectively. Find the linear transformation $T : P_2(R) \to P_3(R)$ related to the matrix $A = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \end{bmatrix}$.

(2) Let $T : R^2 \to R^2, T(x, y) = (x - y)$, $\forall (x, y) \in R^2$ and $B_1 = \{(1, 1), (1, 0)\}$, $B_2 = \{(2, 3), (4, 5)\}$ then find $[T; B_1, B_2]$.

(d) Answer any one of the following : **5**

(1) Let $T:V \to V$ be a linear transformation and B is any basis of V then T is singular if and only if $\det([T;B])=0$.

(2) Find Eigen value for the linear transformation $T:R^2 \to R^2;\ T(a,b)=(3b,2a-b),\ \forall (a,b)\in R^2$.

5 (a) Answer the question briefly : **4**

(1) What is the formula of radius of curvature for function $r=f(\theta)$.

(2) Define point of inflexion.

(3) Define Double Point.

(4) Radius of curvature of circle with radius r _____.

(b) Answer any one of the following : **2**

(1) Show that the curve $y=x^4$ is concave upward at origin.

(2) Find asymptotes parallel to coordinate axis for the curve $x^2y^2=a^2\left(x^2+y^2\right)$.

(c) Answer any one of the following : **3**

(1) Find the radius of curvature of cardioid $r=a(1-\cos\theta)$.

(2) Find the radius of curvature of cardioid $r^2=a^2\cos 2\theta$.

(d) Answer any one of the following : **5**

(1) Obtain the formula for radius of curvature of the curve $y = f(x)$.

(2) Find multiple points on the curve $x^3 + y^3 - 3x^2 - 3xy + 3x + 3y - 1 = 0$ and determine its type.